**Unit 1- Lesson 2- Equivalent Ratios**

**Student Outcomes**

* Students develop an intuitive understanding of equivalent ratios by using tape diagrams to explore possible quantities of each part when given the part-to-part ratio. Students use tape diagrams to solve problems when the part-to-part ratio is given and the value of one of the quantities is given.
* Students formalize a definition of equivalent ratios: Two ratios, $A$:$ B$ and $C$:$ D$, are equivalent ratios if there is a positive number, $c$, such that $C=cA$ and $D=cB$.
* Given a ratio, students identify equivalent ratios. Students use tape diagrams and the description of equivalent ratios to determine if two ratios are equivalent.
* Students relate the positive number, $c$, in the description of equivalent ratios to the tape diagrams they have been using to find equivalent ratios.

**Notes**

Exercise 1 (5 minutes)

This exercise continues to reinforce the student’s ability to relate ratios to the real world, as practiced in Lessons 1 and 2. Provide students with time to think of a one-sentence story problem about a ratio.

Exercise 1

Write a one-sentence story problem about a ratio.

The ratio of the number of sunny days to the number of cloudy days in this town is 3: 1.

Write the ratio in two different forms.

3: 1 3 to 1

Have students share their sentences with each other in pairs or trios. Ask a few students to share with the whole class.

Exercise 2 (15 minutes)

|  |  |
| --- | --- |
| Shanni’s Ribbon | Mel’s Ribbon |
| 7 | 3 |
| 14 | 6 |
| 21 | 9 |

Ask students to read the problem and then describe in detail what the problem is about, if possible, without looking back at the description. This strategy encourages students to really internalize the information given as opposed to jumping right into the problem without knowing the pertinent information.

* Let’s represent this ratio in a table.
* We can use a tape diagram to represent the ratio of the lengths of ribbon. Let’s create one together.

Walk through the construction of the tape diagram with students as they record with you.

* How many units should we draw for Shanni’s portion of the ratio?
	+ *Seven*
* How many units should we draw for Mel’s portion of the ratio?
	+ *Three*

Exercise 2

Shanni and Mel are using ribbon to decorate a project in their art class. The ratio of the length of Shanni’s ribbon to the length of Mel’s ribbon is 7: 3.

Draw a tape diagram to represent this ratio:

Shanni

Mel

* What does each unit on the tape diagram represent?
	+ *Allow students to discuss; they should conclude that they do not really know yet, but each unit represents some unit that is a length.*
* What if each unit on the tape diagrams represents 1 inch? What are the lengths of the ribbons?
	+ *Shanni’s ribbon is 7 inches; Mel’s ribbon is 3 inches.*
* What is the ratio of the lengths of the ribbons?
	+ *7:3 (Make sure that the students feel comfortable expressing the ratio itself as simply the pair of numbers 7:3 without having to add units.)*

*Scaffolding:*

If students do not see that each unit represents a given length, write the length of each unit within the tape diagram units, and have students add them to find the total.

* What if each unit on the tape diagrams represents 2 meters? What are the lengths of the ribbons?
	+ *Shanni’s ribbon is 14 meters; Mel’s ribbon is 6 meters.*
* How did you find that?
	+ *Allow students to verbalize and record using a tape diagram.*
* What is the ratio of the length of Shanni’s ribbon to the length of Mel’s ribbon now? Students may disagree; some may say it is 14:6, and others may say it is still 7:3.
	+ *Allow them to debate and justify their answers. If there is no debate, initiate one:* A friend of mine told me the ratio would be (provide the one that no one said, either 7:3 or 14:6). Is she right?
* What if each unit represents 3 inches? What are the lengths of the ribbons? Record. Shanni’s ribbon is 21 inches; Mel’s ribbon is 9 inches. Why?
	+ *7 times 3 equals 21; 3 times 3 equals 9.*
* If each of the units represents 3 inches, what is the ratio of the length of Shanni’s ribbon to the length of Mel’s ribbon?
	+ *Allow for discussion as needed.*
* We just explored three different possibilities for the length of the ribbon; did the number of units in our tape diagrams ever change?
	+ *No.*
* What did these 3 ratios, 7:3, 14:6, 21:9, all have in common?
	+ *Write the ratios on the board. Allow students to verbalize their thoughts without interjecting a definition. Encourage all to participate by asking questions of the class with respect to what each student says, such as, “Does that sound right to you?”*
* Mathematicians call these ratios *equivalent*. What ratios can we say are equivalent to 7:3?

Shanni and Mel are using ribbon to decorate a project in their art class. The ratio of the length of Shanni’s ribbon to the length of Mel’s ribbon is 7:3.

Draw a tape diagram to represent this ratio:

***7 inches***

***3 inches***

***7:3***

Shanni

Mel

***14 meters***

***6 meters***

***14:6***

Shanni

***2 m***

***2 m***

***2 m***

***2 m***

***2 m***

***2 m***

***2 m***

***2 m***

***2 m***

***2 m***

Mel

***21 inches***

***9 inches***

***21:9***

Shanni

***3 in***

***3 in***

***3 in***

***3 in***

***3 in***

***3 in***

***3 in***

***3 in***

***3 in***

***3 in***

Mel

Exercise 3 (7 minutes)

Work as a class or allow students to work independently first and then go through as a class.

Exercise 3

Mason and Laney ran laps to train for the long-distance running team. The ratio of the number of laps Mason ran to the number of laps Laney ran was 2 to 3.

1. If Mason ran 4 miles, how far did Laney run? Draw a tape diagram to demonstrate how you found the answer.

***4 miles***

***2 mi***

***2 mi***

***Mason***

***Laney***

***6* *miles***

***2 mi***

***2 mi***

***2 mi***

1. If Laney ran 930 meters, how far did Mason run? Draw a tape diagram to determine how you found the answer.

***Mason***

***310***

***310***

***620 meters***

***310***

***310***

***310***

***Laney***

***930 meters***

1. What ratios can we say are equivalent to 2:3?

4:6 and 620:930

Exercise 4 (7 minutes)

Allow students to work the exercise independently and then compare their answers with a neighbor’s answer.

Exercise 4

Josie took a long multiple-choice, end-of-year vocabulary test. The ratio of the number of problems Josie got incorrect to the number of problems she got correct is 2:9.

1. If Josie missed 8 questions, how many did she get right? Draw a tape diagram to demonstrate how you found the answer.

***4***

***4***

***8***

***4 x 9***

***36 right***

***Wrong***

***Right***

1. If Josie missed 20 questions, how many did she get right? Draw a tape diagram to demonstrate how you found the answer.

***10***

***10***

***20***

***10 x 9***

***90 right***

***Wrong***

***Right***

1. What ratios can we say are equivalent to 2:9?

8:36 and 20:90

1. Come up with another possible ratio of the number Josie got wrong to the number she got right.

***5***

***5***

***5 x 9 = 45***

***10:45***

1. How did you find the numbers?

Multiplied 5 x 2 and 5 x 9

1. Describe how to create equivalent ratios.

Multiply both numbers of the ratio by the same number ( any number you choose).

Example 1 (7 minutes)

Present Example 1 by reading it aloud or asking a student to read it aloud. Then encourage students to discuss what would need to be done. Guide the students to a mathematically correct conclusion and have them summarize their decisions.

Conclude by having students come up with the total number of students that would make Jasmine’s statement true.

Example 1

The morning announcements said that two out of every seven 6th graders in the school have an overdue library book. Jasmine said, “That would mean 24 of us have overdue books!” Grace argued, “No way. That is way too high.” How can you determine who is right?

You would have to know the total number of 6th graders, and then see if the ratio 24:total is equivalent to 2:7.

 ***2: 7 24: 84***

***x 12***

***x 12***

* Let’s look at the ratios we determined in Example 1. We found the ratios 2:7 and 24:84.
* How have we previously determined two sets of ratios to be equivalent?
	+ *Each number in the first ratio must be multiplied by the same positive number in order to determine the corresponding numbers in the second ratio.*
* Let’s test these two ratios to see if they are equivalent. Since the corresponding number to 2 in the second ratio is 24, what must we multiply 2 by to find 24?
	+ *12*
* We can determine from this that 12 is the positive number, $c$, that we will multiply each number in the first ratio by to determine the corresponding numbers in the second ratio.
* If we multiply 2 by 12, then following the description, we must also multiply 7 by 12. What is the product of 7 x 12?
	+ *84*
* Is 84 the number that corresponds to 7?
	+ *Yes.*

Allow the students to finish the remaining problems independently.

Allow students to indicate their answers orally for each problem and debate with classmates when there are disagreements. If needed, step in and guide students to the correct reasoning process, ensuring all students come to understand how to use the description to determine equivalence.

Exercise 1 (20 minutes)

Exercise 1

Decide whether or not each of the following pairs of ratios is equivalent.

* If the ratios are not equivalent, find a ratio that is equivalent to the first ratio.
* If the ratios are equivalent, identify the positive number, $c$, that could be used to multiply each number of the first ratio by in order to get the numbers for the second ratio.
	1. 6:11 and 42:88 \_\_ \_ Yes, the value, $c$, is: \_\_ \_

 x  **No, an equivalent ratio would be:**  **42:77**

***× 7***

***× 8***

* 1. 0: 5 and 0: 20 x Yes, the value, $c$, is: 4

***?***

***× 4***

**\_\_ \_ No, an equivalent ratio would be: \_\_\_**

***0 × 4 = 0***

***5 × 4 = 20***

Exercise 2 (8 minutes)

Exercise 2

In a bag of mixed walnuts and cashews, the ratio of number of walnuts to number of cashews is 5:6. Determine the amount of walnuts that are in the bag if there are 54 cashews. Use a tape diagram to support your work. Justify your answer by showing that the new ratio you created of number of walnuts to number of cashews is equivalent to 5:6.

 ***54***

***walnuts***

***cashews***

 **9 9 9 9 9**

 **9 9 9 9 9 9**

54 divided by 6 equals 9.
5 times 9 equals 45.
There are 45 walnuts in the bag.
The ratio of number of walnuts to number of cashews is 45:54. That ratio is equivalent to 5:6.

 5:6 and 45:54

***× 9***

***× 9***

**Closing**

Ask students to share their answers to part (f); then summarize by presenting the following definition:

Two ratios $A:B $and $C:D $are *equivalent* if there is a positive number, $c$, such that $C=cA $and $D=cB$. Ratios are equivalent if there is a positive number that can be multiplied by both quantities in one ratio to equal the corresponding quantities in the second ratio.

*Note:* If students do not have a sufficient grasp of algebra, they should not use the algebraic definition. It is acceptable to use only the second definition.

* How can we use the description of equivalent ratios to find an equivalent ratio?
* What do the numbers in the boxes of the tape diagram represent in terms of the ratios?
	+ *Inside each of the boxes, the positive number, c, comes from the value of one unit in the tape diagram.*
* We can determine that to find an equivalent ratio, the positive number, $c$, must be the same in each box in the tape diagram. This can also be described as “constant.” If the number, $c$, is *constantly* the same number, then the ratios are equivalent. Like in Exercise 4, the value of each unit is 9. It is constantly nine. We multiplied 5 by the *constant* 9 and multiplied the 6 by the *constant* 9 to determine the equivalent ratio.

Lesson Summary

Recall the description:

Two ratios $A:B $and $C:D $are equivalent ratios if there is a positive number, $c$, such that $C=cA $and $D=cB$.

Ratios are equivalent if there is a positive number that can be multiplied by both quantities in one ratio to equal the corresponding quantities in the second ratio.

This description can be used to determine whether two ratios are equivalent.

**Unit 1- Lesson 2- Equivalent Ratios Names:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Partner Practice \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

1. a.Write a one-sentence story problem about a ratio.

b. Write the ratio in two different forms.

2. Shanni and Mel are using ribbon to decorate a project in their art class. The ratio of the length of Shanni’s ribbon to the length of Mel’s ribbon is 7: 3.

Draw a tape diagram to represent this ratio:

3. Mason and Laney ran laps to train for the long-distance running team. The ratio of the number of laps Mason ran to the number of laps Laney ran was 2 to 3.

* 1. If Mason ran 4 miles, how far did Laney run? Draw a tape diagram to demonstrate how you found the answer.

* 1. If Laney ran 930 meters, how far did Mason run? Draw a tape diagram to determine how you found the answer.
	2. What ratios can we say are equivalent to 2:3?

4. Josie took a long multiple-choice, end-of-year vocabulary test. The ratio of the number of problems Josie got incorrect to the number of problems she got correct is 2:9.

* 1. If Josie missed 8 questions, how many did she get right? Draw a tape diagram to demonstrate how you found the answer.
	2. If Josie missed 20 questions, how many did she get right? Draw a tape diagram to demonstrate how you found the answer.
	3. What ratios can we say are equivalent to 2:9?
	4. Come up with another possible ratio of the number Josie got wrong to the number she got right.
	5. How did you find the numbers?
	6. Describe how to create equivalent ratios.

5. Write two ratios that are equivalent to 1:1.

6. Write two ratios that are equivalent to 3:11.

7.

a. The ratio of the width of the rectangle to the height of the rectangle is \_\_\_\_\_\_\_\_ to \_\_\_\_\_\_\_\_\_.

 b. If each square in the grid has a side length of 8 mm, what is the length and width of the rectangle?

8. For a project in their health class, Jasmine and Brenda recorded the amount of milk they drank every day. Jasmine drank 2 pints of milk each day, and Brenda drank 3 pints of milk each day.

a. Write a ratio of number of pints of milk Jasmine drank to number of pints of milk Brenda drank each day.

b. Represent this scenario with tape diagrams.

c. If one pint of milk is equivalent to 2 cups of milk, how many cups of milk did Jasmine and Brenda each drink? How do you know?

d. Write a ratio of number of cups of milk Jasmine drank to number of cups of milk Brenda drank.

e. Are the two ratios you determined equivalent? Explain why or why not.

9. The morning announcements said that two out of every seven 6th graders in the school have an overdue library book. Jasmine said, “That would mean 24 of us have overdue books!” Grace argued, “No way. That is way too high.” How can you determine who is right?

10. Decide whether or not each of the following pairs of ratios is equivalent.

* If the ratios are not equivalent, find a ratio that is equivalent to the first ratio.
* If the ratios are equivalent, identify the positive number, $c$, that could be used to multiply each number of the first ratio by in order to get the numbers for the second ratio.

|  |  |
| --- | --- |
| 11. 6 : 11 and 42 : 8812. 0 : 5 and 0 : 20 | \_\_\_\_ Yes, the value, $c$, is: \_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_ No, an equivalent ratio would be: \_\_\_\_\_\_\_ \_\_\_\_ \_\_\_\_ Yes, the value, $c$, is: \_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_ No, an equivalent ratio would be: \_\_\_\_\_\_\_ \_\_\_\_ |

**13.** In a bag of mixed walnuts and cashews, the ratio of number of walnuts to number of cashews is 5:6. Determine the amount of walnuts that are in the bag if there are 54 cashews. Use a tape diagram to support your work. Justify your answer by showing that the new ratio you created of number of walnuts to number of cashews is equivalent to 5:6.

14. Use diagrams or the description of equivalent ratios to show that the ratios 2:3, 4:6, and 8:12 are equivalent.

15. Prove that 3:8 is equivalent to 12:32.

a. Use diagrams to support your answer.

b. Use the description of equivalent ratios to support your answer.

16. The ratio of Isabella’s money to Shane’s money is 3:11. If Isabella has $33, how much money do Shane and Isabella have together? Use diagrams to illustrate your answer.

**Unit 1- Lesson 2- Equivalent Ratios Name:**

**Exit Ticket/HW**

1. For every two dollars that Pam saves in her account, her brother saves five dollars in his account.

* 1. Determine a ratio to describe the money in Pam’s account to the money in her brother’s account.
	2. If Pam has 40 dollars in her account, how much money does her brother have in his account? Use a tape diagram to support your answer.
	3. Record the equivalent ratio using the information from part b above.
	4. Create another possible ratio that describes the relationship between Pam’s account and her brother’s account.

2. There are 35 boys in the sixth grade. The number of girls in the sixth grade is 42. Lonnie says that means the ratio of number of boys in the sixth grade to number of girls in the sixth grade is 5:7. Is Lonnie correct? Show why or why not.